Graduate Algebra III: Linguistics

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Graduate Colloquium, Jan. 2023

Jokes that can't be put on slides

(when I explain the title of the talk and such)

We'll see two kinds of (mathy?) linguistics

Because

- We have 30 min so I need to talk about more stuff.
- We need to include math so I could stay in this department.

About this talk

- Part I: Two examples of fancy math words appearing in linguistics literature.
- Part II: Two (concrete, maybe too concrete) problems. (Which have nothing to do with part I.)

Fancy math word I: λ calculus

First, fancy linguistics:

some syntactic rules written in:

phrase-structure-rule form	lambda abstraction form
$\mathrm{S} \rightarrow \mathrm{S}$ and S	$S_1' \,\&\, S_2'$
$\mathrm{S} \rightarrow \mathrm{S}$ or S	$S_1' \vee S_2'$
$\mathrm{VP} \rightarrow \mathrm{VP}$ and VP	$\lambda x\,(\mathrm{VP}_1'(x) \,\&\, \mathrm{VP}_2'(x))$
$\mathrm{NP} \rightarrow \mathrm{NP}$ or NP	$\lambda P\,(\mathrm{NP}_1'(P) \vee \mathrm{NP}_2'(P))$

Slightly more complicated (useful) ones

S	→	NP VP	$NP'(VP')$
NP	→	some/a CN	$\lambda P.(\exists x)(CN'(x) \,\&\, P(x))$
NP	→	every CN	$\lambda P.(\forall x)(CN'(x) \rightarrow P(x))$
NP	→	John, Mary, ...	$\lambda P.P(j)$, etc.

Why to write in this way?

It'll be (allegedly) easier to verify equivalences and non-equivalences like:

- Melanie studies math and is on probation — is equivalent to
- Melanie studies math and Melanie is on probation

- Some student took Greek, Hebrew and Kaqchikel and is taking German and Portuguese — is not equivalent to
- Some student took Greek, Hebrew and Kaqchikel . and some student is taking German and Portuguese

Fancy math word II: (look at all those words)

Let D be a binary relation in a model $\mathbf{M} = \langle E, [\![\,]\!] \rangle$ on the sets $A, B, C \subseteq E$; the following list of properties of relations is useful for defining types of determiners

reflexivity:	$D_E AA$
irreflexivity:	$\sim D_E AA$
symmetry:	$D_E AB \rightarrow D_E BA$
asymmetry:	$D_E AB \rightarrow \sim D_E BA$
anti-symmetry:	$D_E AB \,\&\, D_E BA \rightarrow A = B$

Some results

THEOREM 14.3 *The following pairs of statements are equivalent:*

(1) (i) D *is reflexive*

(ii) $A \subseteq B \rightarrow D\,AB$

(2) (i) D *is irreflexive*

(ii) $D\,AB \rightarrow A \not\subseteq B$

Proofs:

(1) (i) $\rightarrow$ (ii) Assume D is reflexive, and take $A \subseteq B$. Then DAA and since $A \cap B = A$ also $DA(A \cap B)$. By Conservativity then DAB.

(1) (ii) $\rightarrow$ (i) follows directly from the reflexivity of inclusion.

(2) D is irreflexive iff $\sim D$ is reflexive. By (1) D is irreflexive iff $A \subseteq B \rightarrow \sim D\ AB$. Contraposition and double negation gives $D\,AB \rightarrow A \not\subseteq B$.

More

(3) (i) D *is antisymmetric*

(ii) $D\,AB \rightarrow A \subseteq B$

Half of the proof:

(3) (i) $\rightarrow$ (ii). Assume D is antisymmetric and take for some domain E, sets $A, B \subseteq E$ such that $D_E\,AB$. By Conservativity $D_E\,A(A \cap B)$. Extend E to E', and take $A' \subseteq E'$ such that $|A'| = |A|$ and $A \cap A' = A \cap B$. Then $D_E\,A(A \cap A')$ and by Extension $D_{E'}\,A(A \cap A')$. Conservativity gives $D_{E'}\,A\,A'$. Now consider a permutation π of E' which yields identical sets for $A \cap A'$ and $E' - (A \cup A')$ but permutes $A - A'$ with $A' - A$. Quantity gives us $D_{E'}\pi(A)\pi(A')$, i.e., $D_{E'}\,A'\,A$. So we have now $D_{E'}\,A\,A'$ and $D_{E'}\,A'\,A$, and antisymmetry of D gives $A = A'$. Since $A \cap A' = A \cap B$ it follows that $A \cap B = A$, so $A \subseteq B$.

Problems! Finally.

- Problem I: Problem 1 from 1st IOL
- Problem II: Problem 1 (partial) from 13th IOL

IOL: International Linguistics Olympiad

(Which is for little kids, I know.)

Problem I

In 1916 the Russian scholar Jacob Linzbach invented a universal writing system, which he thought should be understandable to all people, regardless of their native tongue. Linzbach called his new language 'Transcendental Algebra'.

Several sentences have been written in Linzbach's language and translated into English:

1. $(\frac{\dot{\Lambda}\dot{\Delta}\dot{\imath}\dot{\Delta}}{\dot{\Delta}\dot{\imath}\dot{\Delta}} + \frac{\dot{\imath}\dot{\Delta}}{\dot{\Delta}})^{\leqslant}$ — The father and the brother are talking.
2. $\mathrm{n}(> \dot{\mathrm{I}})^{[\vdash - \mathrm{t}}$ — The giants are working without haste.
3. $(\frac{\dot{\imath}\dot{\Delta}(-\dot{\Lambda}\dot{\Delta})}{(-\dot{\Lambda}\dot{\Delta})})^{\not{/}} = \boxtimes$ — The orphans are writing a letter.
4. $(-\mathrm{n}\dot{\mathrm{I}}_1)^{\not{/}} - \mathrm{t} = \dot{\mathrm{I}}_2$ — It wasn't us who wrote about you (sg.).
5. $\boxtimes^{\sqrt{\not{/}}} - \mathrm{t} = -\dot{\Delta}_3$ — It was not by her that the letter was written.
6. $(\frac{\dot{\Lambda}\dot{\Delta}\dot{\imath}\dot{\Delta}}{\dot{\Delta}\dot{\imath}\dot{\Delta}})^{-\smile} = [\vdash$ — The father doesn't like the work.
7. $((> \dot{\mathrm{I}}) - \smile)^{\frown} - \mathrm{t} = \frac{\dot{\Lambda}\dot{\Delta}\dot{\imath}\dot{\Delta}}{\dot{\imath}\dot{\Delta}}$ — The wicked giant ate the parents.
8. $\dot{\Delta}_3^{-\mathrm{t}}$ — She is not in a hurry.

Problem I

I'll do it on the board!

Assignment 1. Translate into English:

9. $\dot{I}_3^{\heartsuit - \sqrt{\heartsuit}}$
10. $(\frac{\dot{\Lambda}\dot{\Delta}i\dot{\Delta}}{\dot{\Lambda}\dot{\Delta}i} - \measuredangle)^{\not{/}} + t = \frac{\dot{\Lambda}\dot{\Delta}i\dot{\Delta}}{\dot{\Delta}i\dot{\Delta}} + \frac{\dot{\Lambda}\dot{\Delta}i\dot{\Delta}}{\dot{\Lambda}i\dot{\Delta}}$
11. $\dot{\Delta}_2^{\square - + t - \measuredangle} - t$
12. $\boxtimes^{\sqrt{\cap}} - t = \frac{i\dot{\Delta}}{\dot{i}} - \cap$

Strategies (might work) in (math) olympiads:

- Wild guesses
- Wishful thinking
- Write down "observations" no matter how obvious you think they are

Problem I

I'll do it on the board!

Assignment 2. Write in 'Transcedental Algebra':

13. It wasn't about them that my husband and I (*say:* I and the husband) talked.
14. The people are working reluctantly.
15. The good widow loves the unemployed dwarf.
16. You (pl.) will be talked about.

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Problem II

$$
\begin{aligned}
\textit{mahtlactli-on-cë} \times \textit{mahtlactli} &= \textit{mäcuïl-pöhualli-om-mahtlactli} && (1)\\
\textit{cem-pöhualli} \times \textit{öme} &= \textit{öm-pöhualli} && (2)\\
\textit{yë-pöhualli-on-chicöme} + \textit{mahtlactli-on-nähui} &= \textit{näuh-pöhualli-on-cë} && (3)\\
\textit{mäcuïlli} + \textit{öme} &= \textit{chicöme} && (4)\\
\textit{mahtlactli-om-ëyi} \times \textit{ëyi} &= \textit{cem-pöhualli-on-caxtölli-on-nähui} && (5)\\
\textit{mäcuïlli} \times \textit{ëyi} &= \textit{caxtölli} && (6)
\end{aligned}
$$

Problem II

I'll do it on the board!

(a) Write the equalities (1–6) in numerals.

(b) Write out in Nahuatl: 42; 494.

Recall

- A ring is a set with + and ·.
- $\mathbb{Z}$ is a ring.

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- Wishful thinking
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References

1. Barbara H. Partee , Alice Meulen , Robert E. Wall, *Mathematical Methods in Linguistics* (1993), Springer.
2. *International Linguistics Olympiad*. https://ioling.org/. Accessed Jan. 24, 2023.
3. David S. Dummit, Richard M. Foote, *Abstract Algebra* (2003), Wiley.